# Models of Set Theory II - Winter 2017/2018

Prof. Dr. Peter Koepke, Ana Njegomir Problem sheet 13

**Definition.** A *Ramsey ultrafilter* is an ultrafilter $\mathcal{U} \subseteq [\omega]^\omega$ which contains all co-finite subsets of $\omega$ and such that for every *colouring* $\pi : [\omega]^2 \to 2$ there is $x \in \mathcal{U}$ such that $\pi \upharpoonright [x]^2$ is constant.

**Problem 1** (5 points)**.** Let $\mathbb{U}$ denote the forcing $\langle [\omega]^\omega, \subseteq^*, \omega \rangle$ and let $\pi : [\omega]^2 \to 2$. Prove that $D_\pi = \{x \in [\omega]^\omega \mid \pi \upharpoonright [x]^2 \text{ is constant}\}$ is dense in $\mathbb{U}$.

**Problem 2** (5 points)**.** Let $\mathbb{U}$ denote the forcing $\langle [\omega]^\omega, \subseteq^*, \omega \rangle$. Show that if $G$ is $M$-generic for $\mathbb{U}$, then $G$ is a Ramsey ultrafilter.

**Problem 3** (5 points)**.** If $\mathfrak{p} = \mathfrak{c}$, then there exists a Ramsey ultrafilter.

**Problem 4** (5 points)**.** Let $\mathbb{P}$ be a forcing notion. Show that if $\mathbb{P}$ preserves cofinalities, then it preserves cardinalities.

Please hand in your solutions on Monday, January 22 before the lecture.